Written Exam for the B.Sc. or M.Sc. in Economics winter 2015-16

Micro III

Final Exam

Date: 5 February 2016

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam question consists of 4 pages in total (including the front page)

PLEASE ANSWER ALL QUESTIONS.
PLEASE EXPLAIN YOUR ANSWERS.

1. (a) Find all the pure and mixed-strategy Nash Equilibria of the following game.

		Player 2		
		t_1	t_2	t_3
Player 1	s_1	$1, 0$	$5, 2$	$1, 5$
	s_2	$3, 3$	$2, 1$	$0, 2$

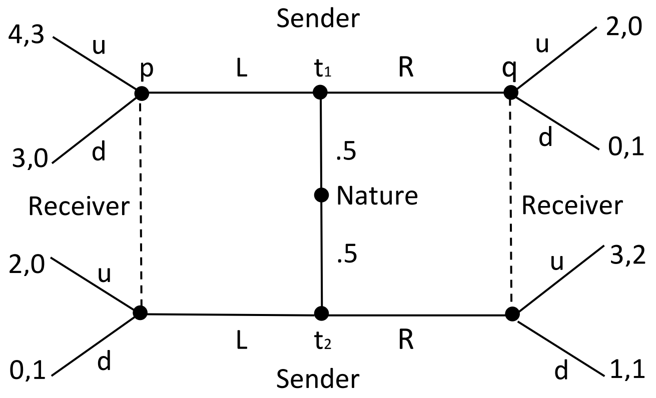
   (b) Suppose now that we introduce a new strategy for Player 1. Denote the corresponding game by G:

		Player 2		
		t_1	t_2	t_3
Player 1	s_1	$1, 0$	$3, 2$	$1, 5$
	s_2	$3, 3$	$2, 1$	$0, 2$
	s_3	$0, 4$	$10, 10$	$0, 11$

   Use iterated elimination of strictly dominated strategies to simplify the game. Explain briefly each step (1 sentence). What is the set of pure and mixed-strategy Nash Equilibria of G?

   (c) Now suppose we repeat G twice. Denote the resulting game by $G(2)$. How many proper subgames are there (not counting the game itself)? Show that there is a Subgame-perfect Nash Equilibrium of $G(2)$ in which (s_3, t_2) is played in stage 1. Be careful to write up the equilibrium.

2. **Signaling.** Consider the following signaling game.



   (a) Find all the (pure strategy) separating Perfect Bayesian Equilibria (PBE).

   (b) Find the (pure strategy) pooling equilibrium in which both types send message L. Does it satisfy signaling requirement 5 (SR5)?

   (c) Explain in your own words the logic behind SR5. You may use the above game as an example.

3. Consider a **second-price sealed bid auction** with two bidders, who have valuations v_1 and v_2, respectively.

   (a) First, assume that the values are distributed independently uniformly with

   $$v_i \sim u(1, 2).$$

   Thus, the values are **private**. Show that there is a symmetric Bayesian Nash Equilibrium where the players bid their valuation: $b_i(v_i) = v_i$ (recall that the auction format is second-price sealed bid).

   (Hint: Look at whether the players can profitably deviate by bidding higher or lower.)

   (b) Consider now the following **common value** setting. The auction format is still *second price.* Each player i observes a signal s_i, where

   $$s_i \sim u(1, 2).$$

   The valuation of the players is the sum of the two signals: for each i,

   $$v_i = s_1 + s_2.$$

   The expected valuation of player i conditional on s_i is $\mathbb{E}[v_i|s_i] = \mathbb{E}[s_1+s_2|s_i] = s_i+\frac{3}{2}$. Suppose players bid their expectation, i.e. that $b_i(s_i) = s_i + \frac{3}{2}$. What is the expected value of player i conditional on s_i and conditional on winning the auction? I.e., what is $\mathbb{E}[v_i|s_i, i \text{ wins}]$?

   (c) Relate your answer in the last question to the concept of the *winner's curse.*

4. Consider the following exercise in which a buyer and a seller have valuations v_b and v_s, but only the seller knows the valuations. The buyer makes an offer of a price, and the seller chooses whether to accept. The details are as follows.

   Valuations. The seller's valuation is uniformly distributed on the unit interval. I.e.

   $$v_s \sim u(0, 1).$$

   The buyer's valuation is $v_b = k \cdot v_s$, where $k > 1$ is common knowledge.

   Information. Seller knows v_s (and hence v_b) but the buyer does **not** know v_b (or v_s).

   Buyer. The buyer makes a single offer, p, which the seller either accepts ($a = 1$) or rejects ($a = 0$). (I.e., it is the *buyer* who sets the price, and seller who decides whether he accepts or rejects.) The buyer gets payoffs

   $$u_b(p, a) = \begin{cases} v_b - p \text{ if } a = 1 \text{ (seller accepts)}, \\ 0 \text{ if } a = 0 \text{ (seller rejects)}. \end{cases}$$

   The buyer's strategy is just a choice of p, since he cannot condition his choice on v_b.

Seller. The seller's payoffs are

$$u_s(p, a) = \begin{cases} p \text{ if } a = 1 \text{ (seller accepts)}, \\ v_s \text{ if } a = 0 \text{ (seller rejects)}. \end{cases}$$

His strategy can be described as a function $a(p, v_s)$, where $a(p, v_s) = 1$ corresponds to accepting the offer of p when his valuation is v_s, and $a(p, v_s) = 0$ corresponds to rejecting it. Suppose that whenever he is indifferent, he accepts the offer.

We will look for a Perfect Bayesian Equilibrium (PBE).

(a) Show that in a PBE, $a^*(p, v_s) = 1$ if and only if $v_s \leq p$.

(b) Buyer's expected payoff from making an offer of p is

$$\pi(p)(\mathbb{E}[v_b|\text{seller accepts}, p] - p),$$

where $\pi(p) = \mathbb{P}(\text{seller accepts}|p)$.

  i. Find $\pi(p)$ given $a^*(p, v_s)$.
  ii. Find $\mathbb{E}[v_b|\text{seller accepts}, p]$ given $a^*(p, v_s)$.

(c) What is the PBE when $k > 2$? What is the probability that trade takes place? How would the answer change if $k < 2$?